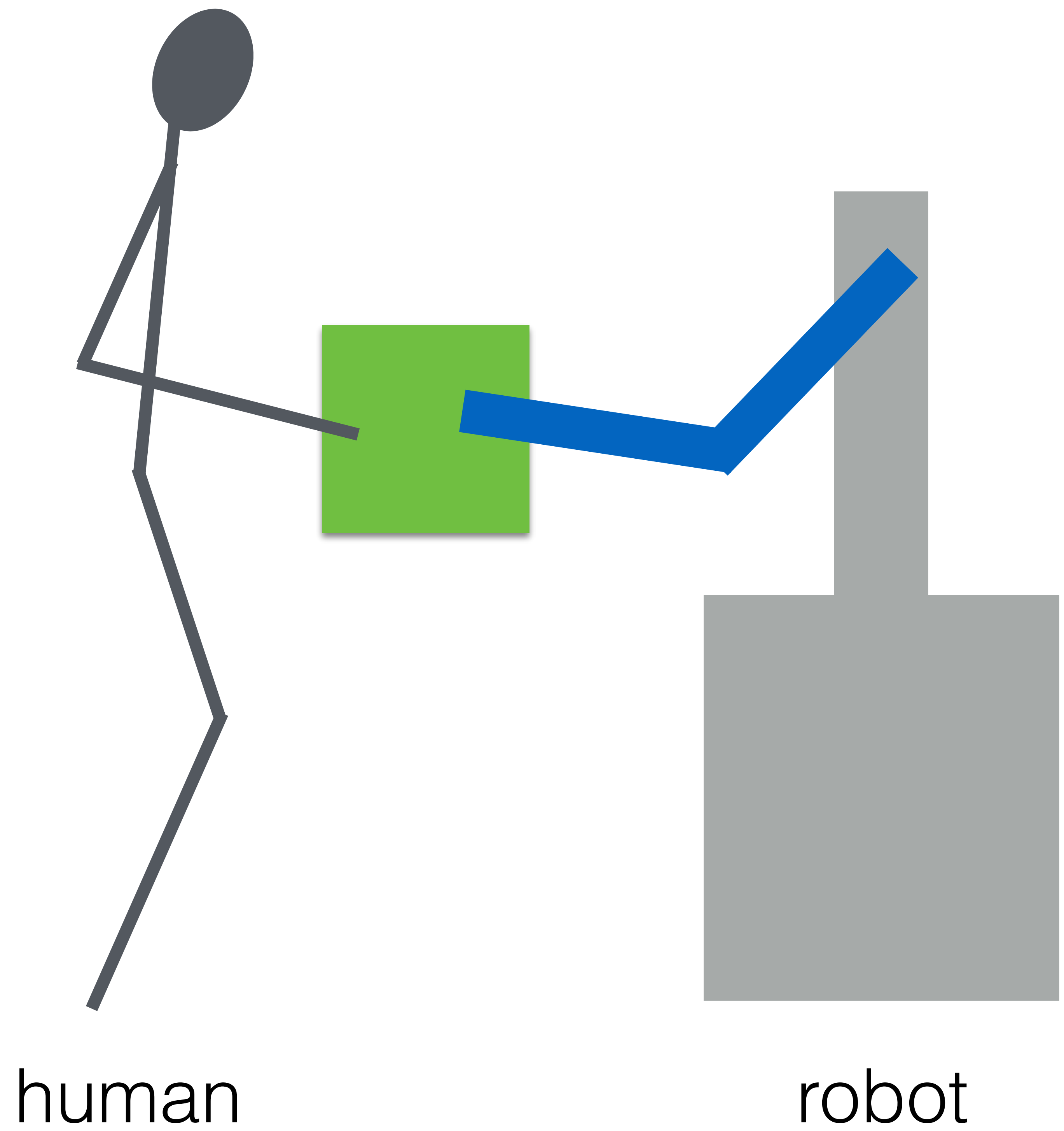
# A Tutorial on Impedance Control and Physical Human-Robot Interaction

Michael Mistry
Reader in Robotics

The 2nd UK Robot Manipulation Workshop
11/7/17

THE UNIVERSITY of EDINBURGH

# Physical Human-Robot Interaction:



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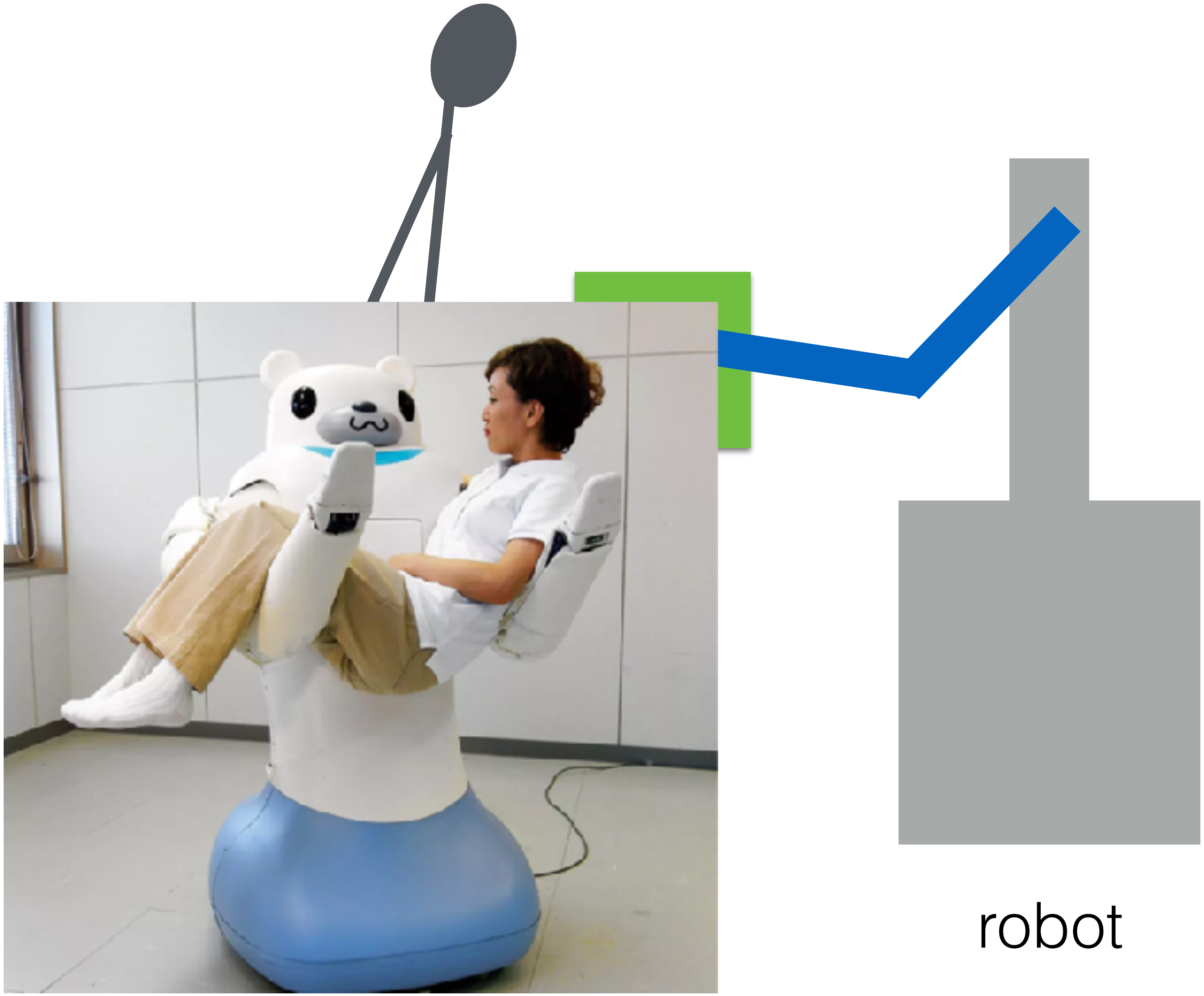
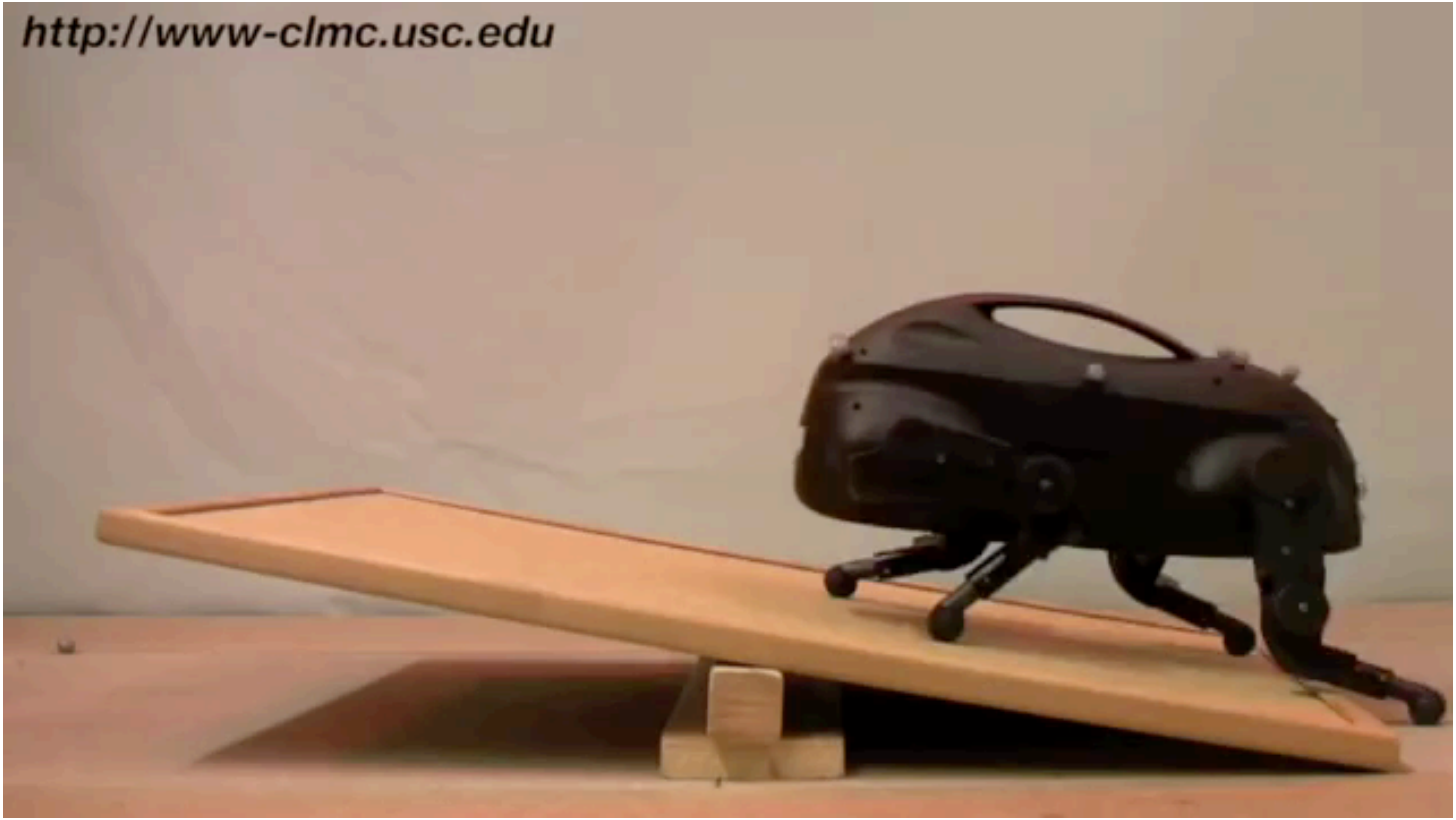


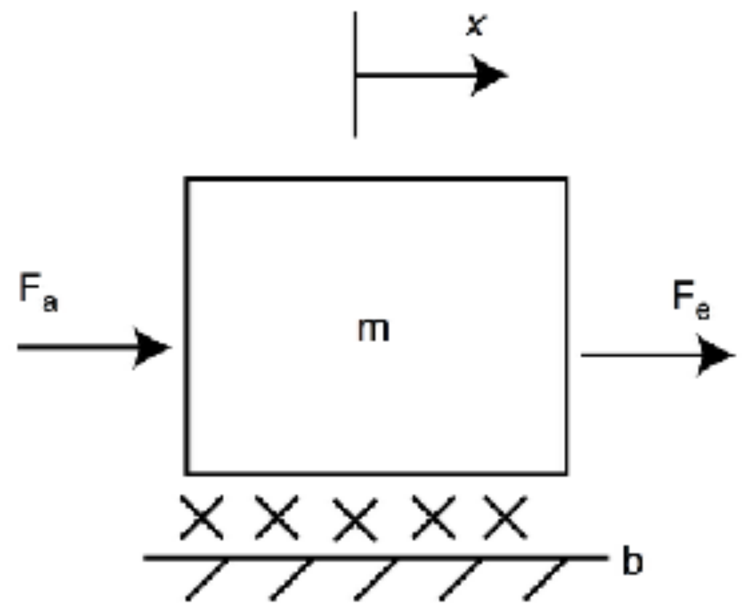

**We need safe interaction!**

# Interacting with an uncertain environment




Kalakrishnan, M., J. Buchli, P. Pastor, M. Mistry, and S. Schaal. 2010. “Fast, Robust Quadruped Locomotion over Challenging Terrain.” In *2010 IEEE International Conference on Robotics and Automation*, 2665–70. ieeexplore.ieee.org.

# A stable controller in isolation, may not be stable in interaction




$$m\ddot{x} + b\dot{x} = F_a + F_e$$ **equation of motion**

$$(ms^2 + bs)X = F_a + F_e$$ **equation of motion (Laplace)**

$$F_a = K_P(R - X) + \frac{K_I}{s}(R - X)$$ **PI Controller**

Hogan, Neville, and Stephen P. Buerger. 2005. "Impedance and Interaction Control." *Robotics and Automation Handbook*

**closed loop transfer function**

$$\frac{X}{R} = \frac{K_P s + K_I}{ms^3 + bs^2 + K_P s + K_I}$$

**closed loop system is stable provided:**

$$K_I < \frac{bK_P}{m}$$

Hogan, Neville, and Stephen P. Buerger. 2005. "Impedance and Interaction Control." *Robotics and Automation Handbook*

**closed loop transfer function**

$$\frac{X}{R} = \frac{K_P s + K_I}{m s^3 + b s^2 + K_P s + K_I}$$

**closed loop system is stable provided:**

$$K_I < \frac{b K_P}{m}$$

**however, interacting with another mass may cause instability!**

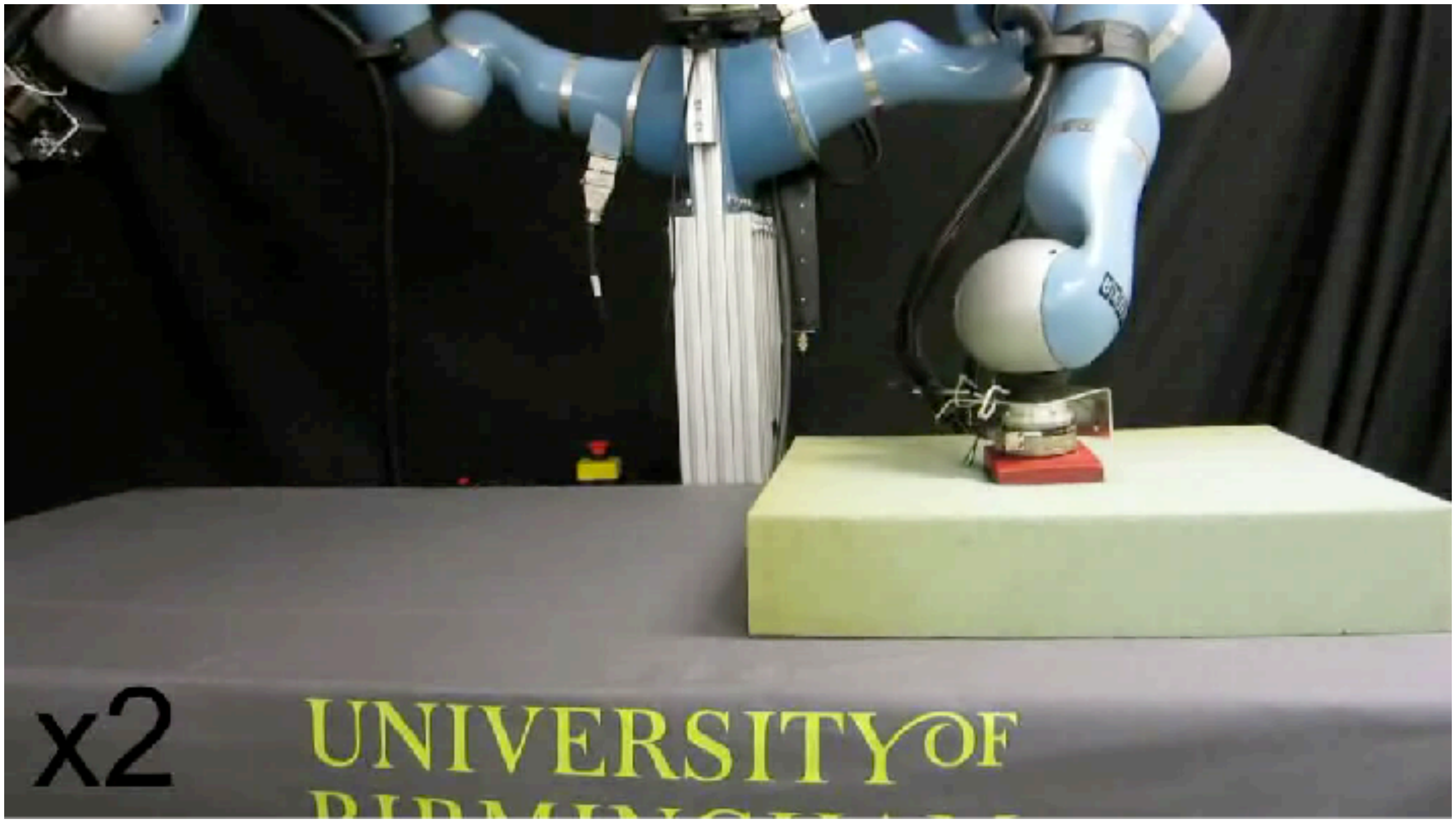
$$K_I < \frac{b K_P}{m + m_2}$$

Hogan, Neville, and Stephen P. Buerger. 2005. "Impedance and Interaction Control." *Robotics and Automation Handbook*

# How to create smarter controllers that can cope with environmental uncertainty?

**How to create smarter controllers that can cope with environmental uncertainty?**

**Try to learn the environment's dynamics?**




M Azad, V Ortenzi, H Lin, E Rueckert, M Mistry, Model Estimation and Control of Compliant Contact Normal Force, Humanoids 2016

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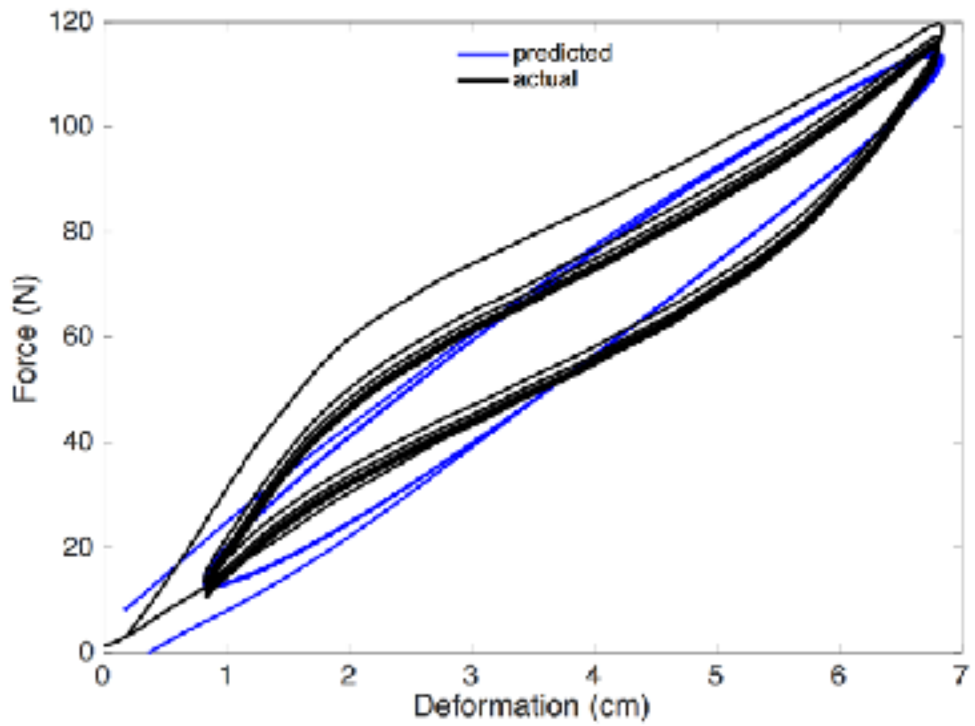
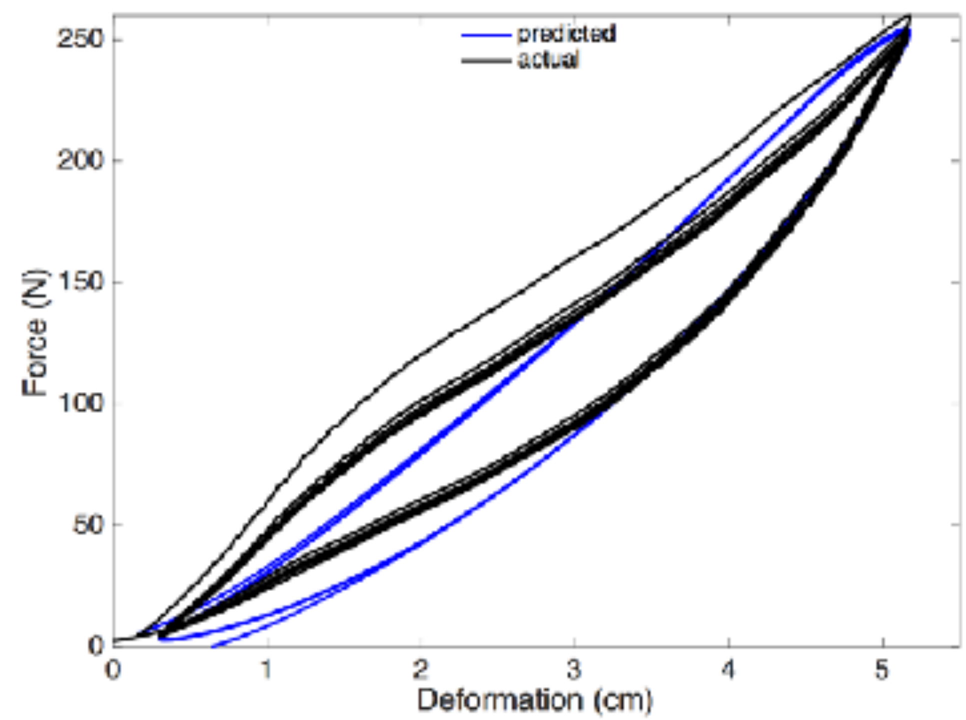


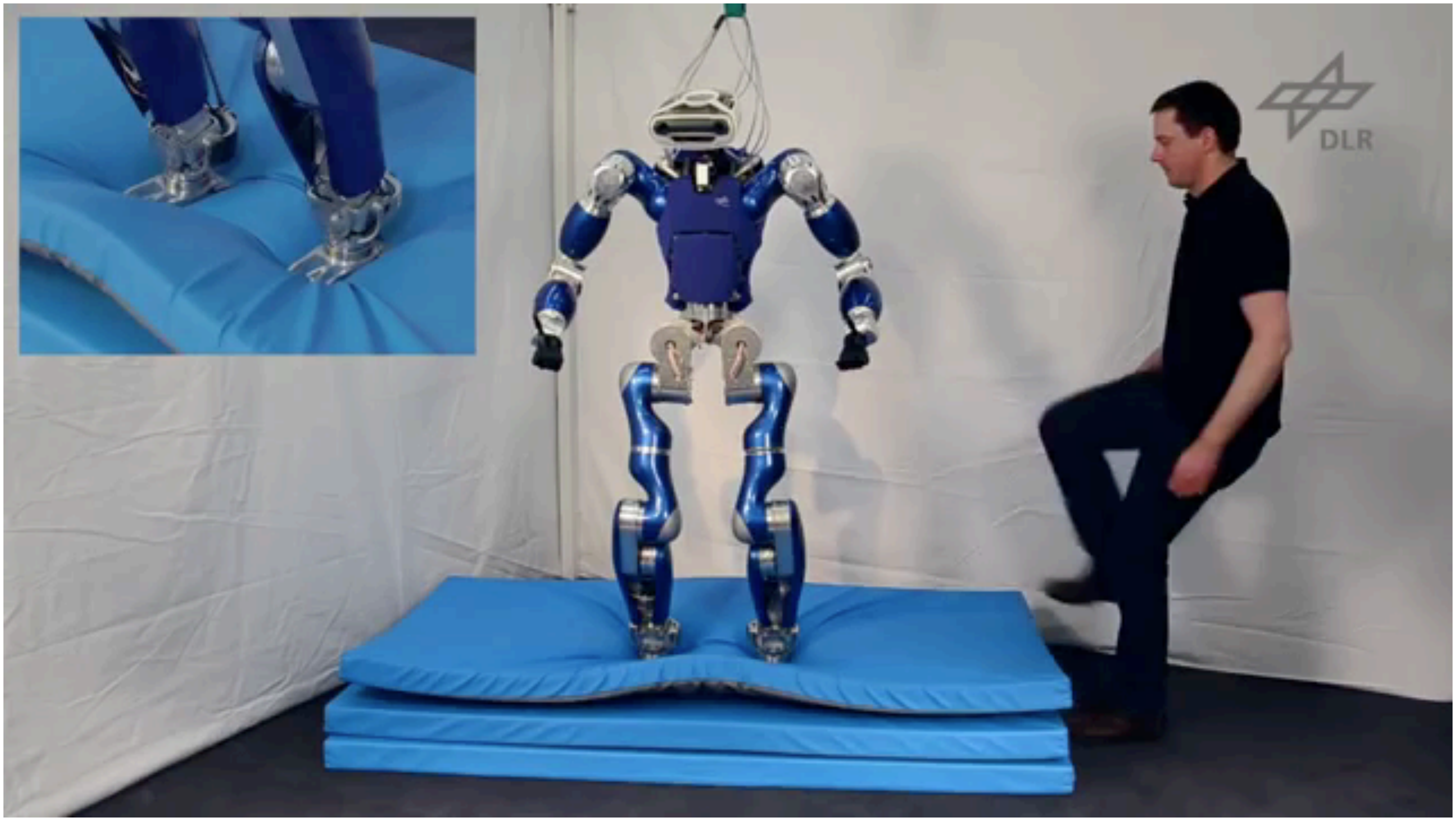




**But stable interaction is required in order to identify the environment!**

M Azad, V Ortenzi, H Lin, E Rueckert, M Mistry, Model Estimation and Control of Compliant Contact Normal Force, Humanoids 2016

How to create smarter controllers that can cope with environmental uncertainty?

“Robust” control: Treat the environment as a disturbance to be rejected?

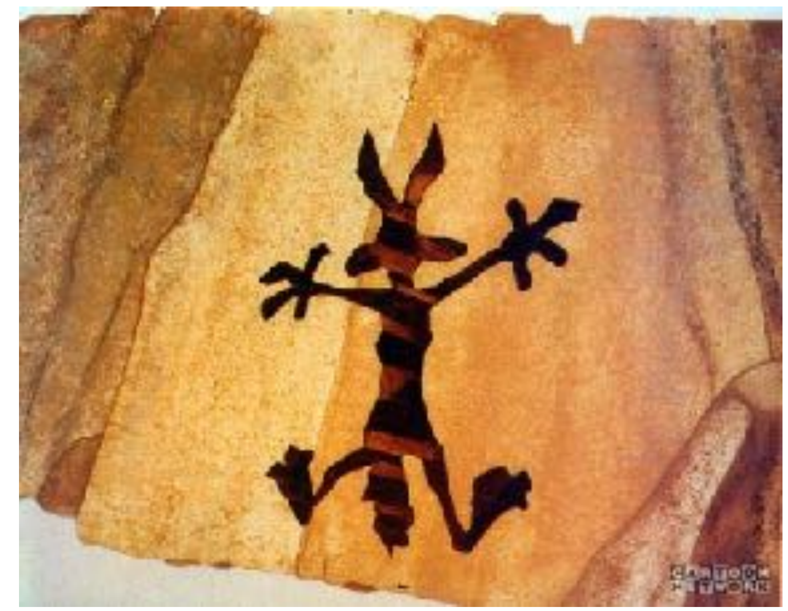



Henze, Bernd, Máximo A. Roa, and Christian Ott. "Passivity-based whole-body balancing for torque-controlled humanoid robots in multi-contact scenarios." *The International Journal of Robotics Research* 35.12 (2016): 1522-1543.

**How to create smarter controllers that can cope with environmental uncertainty?**

**“Robust” control: Treat the environment as a disturbance to be rejected?**

**But environmental forces may be unbounded:**

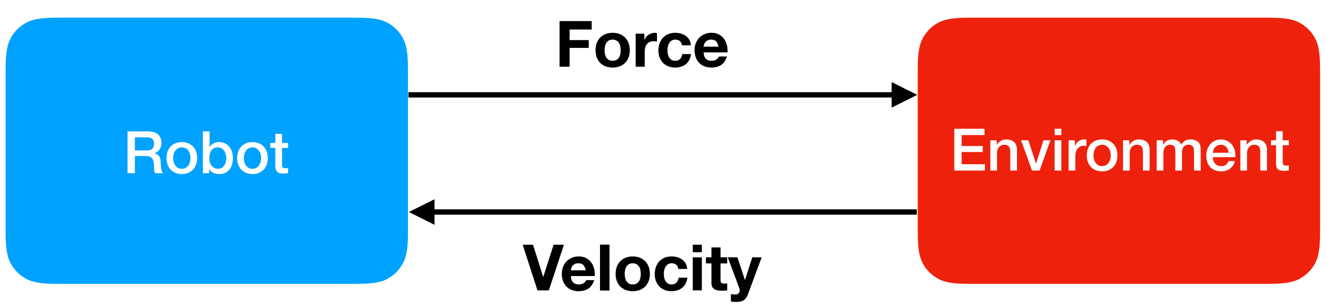


**Gauss’s principle of least constraint: The mountain will apply exactly the amount of force required to prevent your motion through it. No more, and no less.**

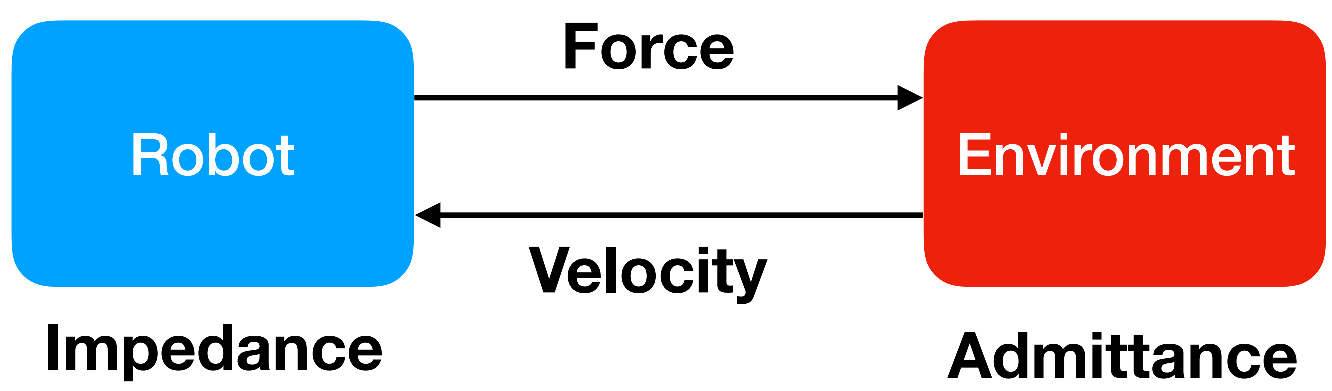
**Furthermore, the “robust” controller still needs to assume some structure of the uncertainty (and then may only be able to cope with disturbances compatible with that structure)**

**How to create smarter controllers that can cope with environmental uncertainty?**

**Consider the *Energy* involved in the interaction:**




**Interaction means a *Transfer of Energy*,**
**at a rate of Force*Velocity**




**An *Impedance* is a dynamic operator that determines an output effort (force) given an input flow (velocity).**

**e.g. like a spring**

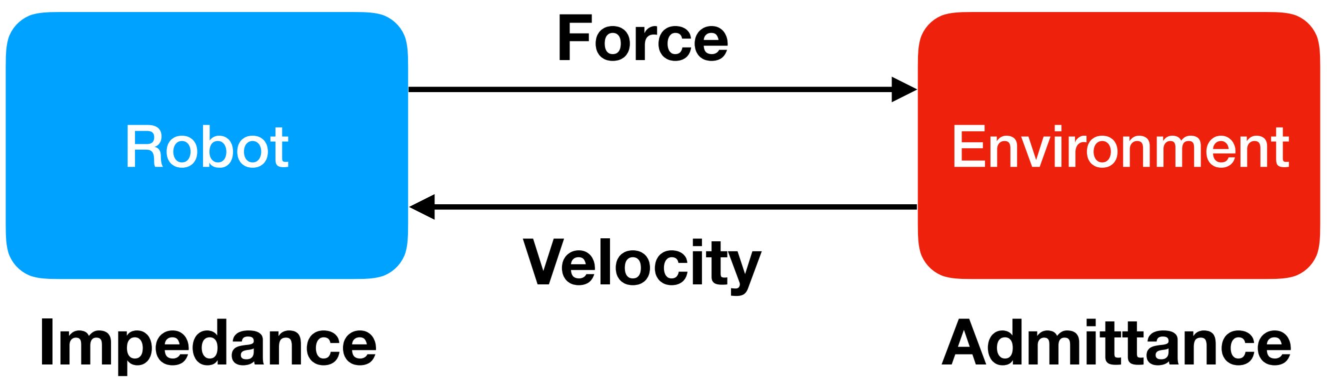
**An *Admittance* is a dynamic operator that determines an output flow (velocity) given an input effort (force).**

**e.g. like a mass, or a wall!**

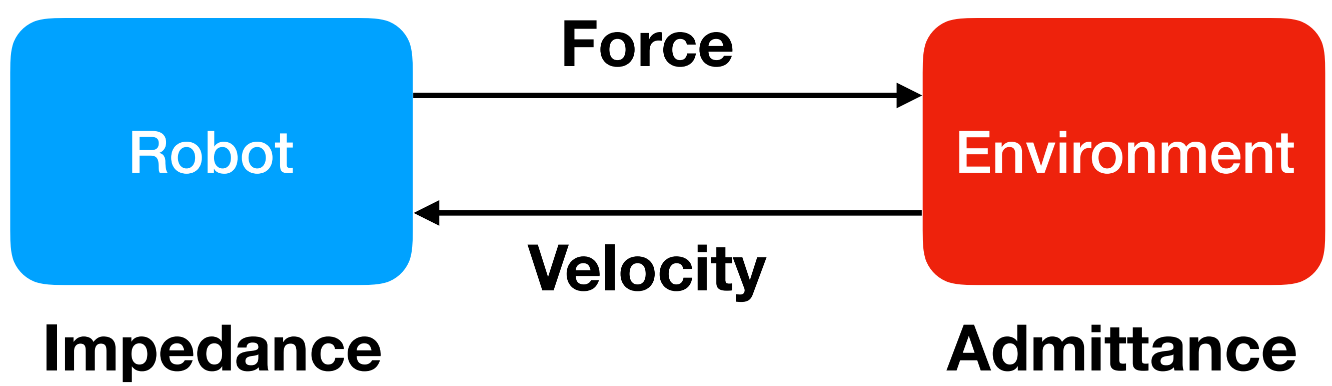
**A dynamic element can only be either an impedance or an admittance. This means it can never simultaneously determine its own force and velocity!**

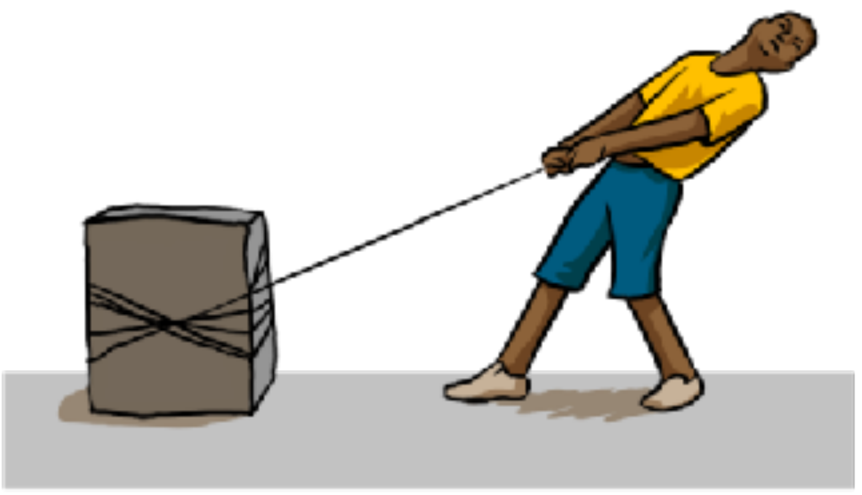
**An Impedance can only interact with an Admittance, and vice versa.**

**When two agents physically interact, one must dictate motion (Admittance), while the other dictates force (Impedance).**




**Why do I label the robot as impedance and environment as admittance?**



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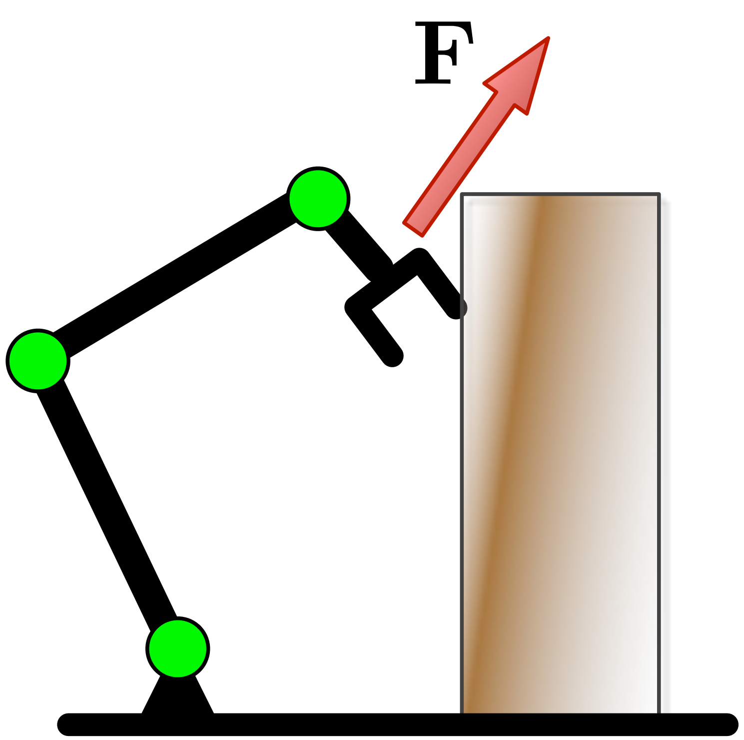



A robot can rarely impose a velocity on an environment: E.g. a heavy object may move slower than we like, a rigid wall may never move.

Rather, the robot imposes a force on the environment, and then only after, motion determined (and could be zero)

Thus it makes sense to treat the environment as an Admittance.

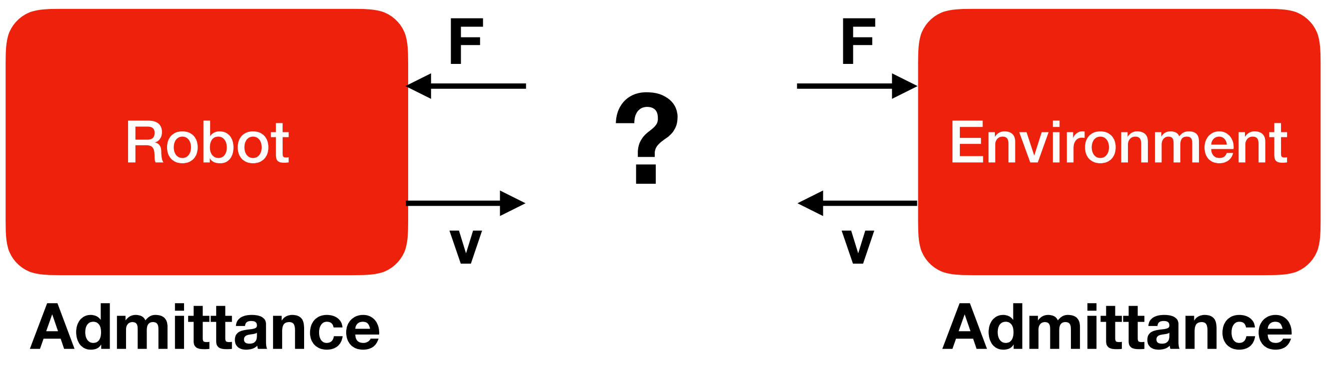
As a consequence, the robot must be an Impedance

## What about traditional position-controlled manipulators?

Kuka industrial robots

Typically have high inertias (due to high mass and high gear ratios)

These are in fact admittance-controlled devices.

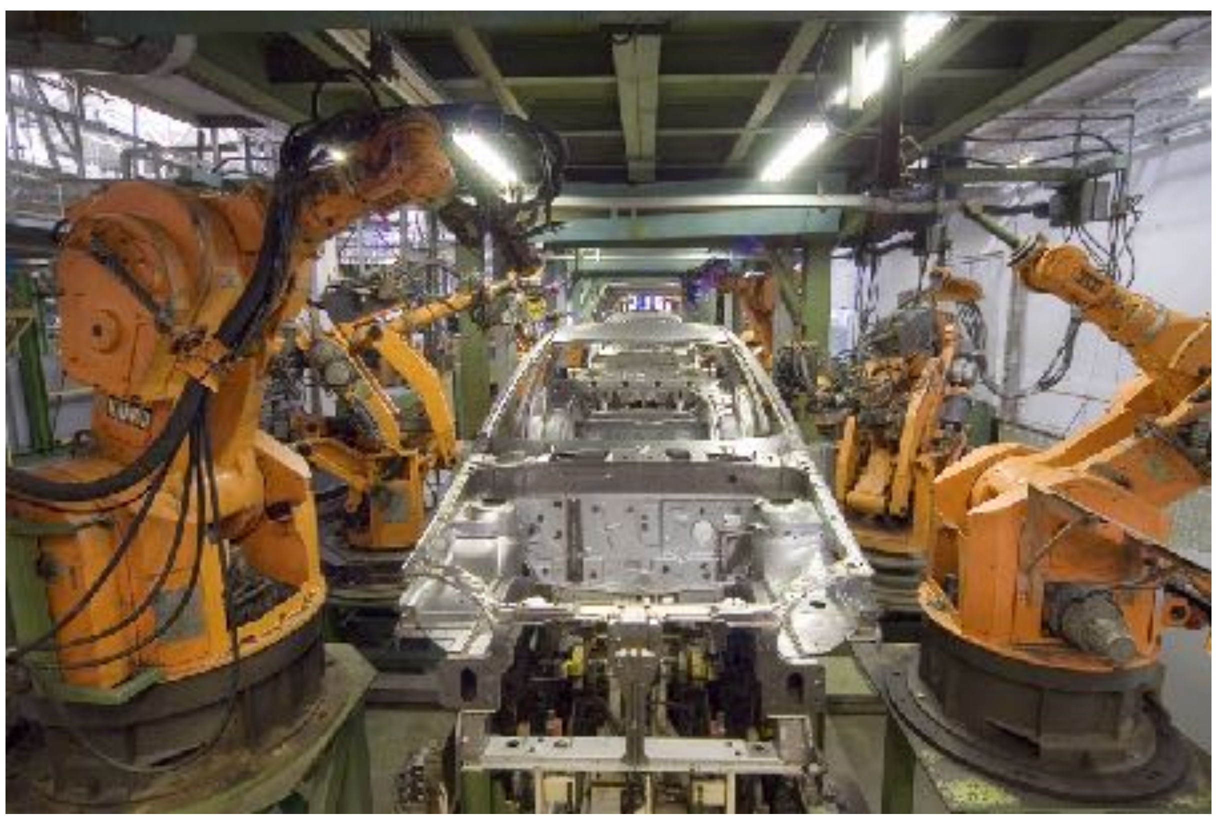



Incompatible?

If no interaction, than no problem!

However, interaction may be dangerous due to high inertia.
Also requires force sensing at contact (which introduces delay)

However, can create very stiff interaction (which may be desirable for some haptic applications)

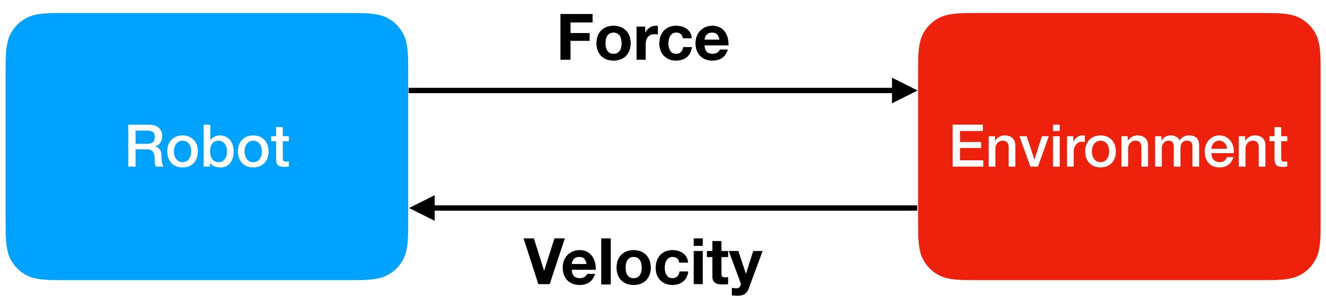


Kuka industrial robots

Moog HapticMaster

**How to create smarter controllers that can cope with environmental uncertainty?**

**If we consider the *Energy* involved in the interaction:**




**Can we create a controller that only dissipates energy, and never generates it?**

**How to create smarter controllers that can cope with environmental uncertainty?**

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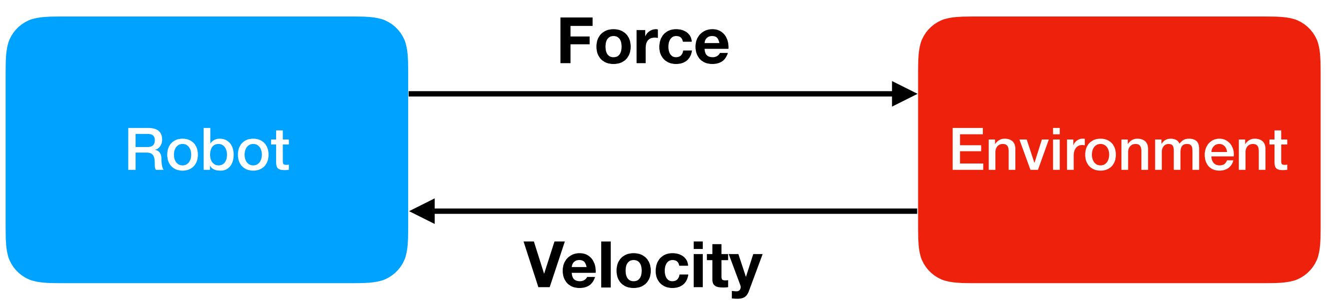


**Can we create a controller that only dissipates energy, and never generates it?**

**Passivity: notion that the system cannot, for any time period, output more energy at its port of interaction than has in total been put into the same port for all time.**

**Examples of Passive Mechanical Elements (and their Electrical equivalents) :**

| | |
|---|---|
| **Spring** | **Capacitor** |
| **Mass** | **Inductor** |
| **Damper** | **Resistor** |

Hogan, Neville, and Stephen P. Buerger. 2005. "Impedance and Interaction Control." *Robotics and Automation Handbook*

# Passivity

**Some important characteristics:**

- **System is Passive if and only if it cannot produce energy.**
- **Output energy is limited to the initial plus accumulated energy in the system**
- **Two passive systems can be combined to form a new passive system.**
- **Feedback connection of two passive systems is stable!**
- **A Non-Passive system interacting with a Passive one, may be destabilised.**

Stramigioli, Stefano. 2015. "Energy-Aware Robotics." In *Mathematical Control Theory I*, edited by M. Kanat Camlibel, A. Agung Julius, Ramkrishna Pasumarthy, and Jacquelien M. A. Scherpen, 37–50. Lecture Notes in Control and Information Sciences 461. Springer International Publishing.

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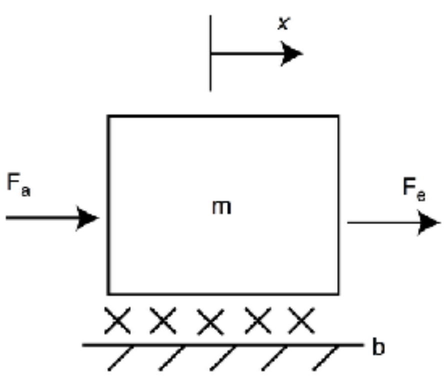
**recall example from earlier:**




$$K_I < \frac{bK_P}{m + m_2}$$

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**Passivity is key for safe and stable interaction!**

**If a controller and robot are passive, model uncertainty can decrease "performance" but never compromise passivity and safety!**

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## How to implement a passive controller:

### Hogan 1985 "Simple" Impedance Control:

joint space: $$\tau = \mathbf{K}_d(\mathbf{q}_d - \mathbf{q}) + \mathbf{D}_d(\dot{\mathbf{q}}_d - \dot{\mathbf{q}})$$

**simple to implement (just PD control)**
**Passive! (note: Integral control is not passive)**
**Only position/velocity sensors required, no force sensors**

Hogan, Neville. 1985. "Impedance Control: An Approach to Manipulation: Part II—Implementation." *Journal of Dynamic Systems, Measurement, and Control* 107 (1). American Society of Mechanical Engineers: 8–16.

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**Only position/velocity sensors required, no force sensors**

task space: $$\dot{\mathbf{x}} = \mathbf{J}\dot{\mathbf{q}}$$

$$\tau = \mathbf{J}^T \left(\mathbf{K}_d(\mathbf{x}_d - \mathbf{x}) + \mathbf{D}_d(\dot{\mathbf{x}}_d - \dot{\mathbf{x}})\right)$$

**still passive!**
**singularity robust (Jacobian is not inverted)**
**but we will still need to take care of redundancy**
**(if the manipulator is redundant)**

Hogan, Neville. 1985. "Impedance Control: An Approach to Manipulation: Part II—Implementation." *Journal of Dynamic Systems, Measurement, and Control* 107 (1). American Society of Mechanical Engineers: 8–16.

# Operational Space Control (Khatib, 1987)

Rigid Body dynamics: $\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{h}(\mathbf{q}, \dot{\mathbf{q}}) = \tau$

Task: $\mathbf{x} = f(\mathbf{q})$

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}$$

We can *decouple* task and null-space forces:

$$\tau = \tau_{\text{task}} + \tau_{\text{null}}$$

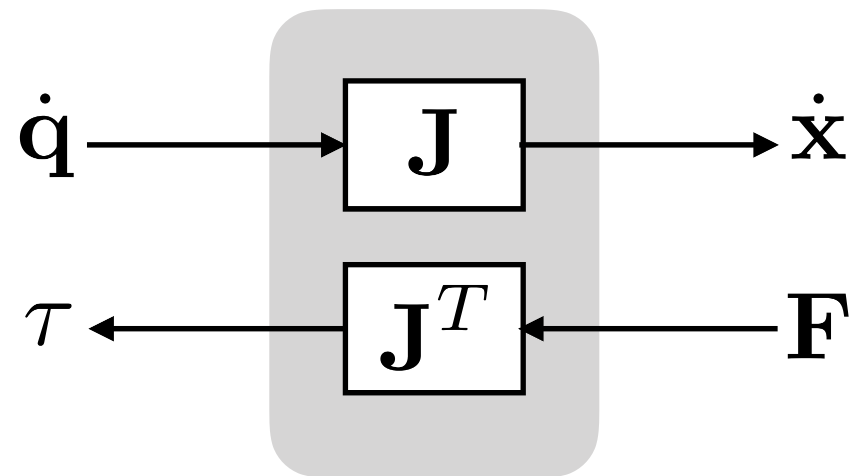
$$\tau = \mathbf{J}^T \bar{\mathbf{J}}^T \tau + \left(\mathbf{I} - \mathbf{J}^T \bar{\mathbf{J}}^T\right) \tau$$

$$\bar{\mathbf{J}} = \mathbf{M}^{-1}\mathbf{J}^T \left(\mathbf{J}\mathbf{M}^{-1}\mathbf{J}^T\right)^{-1}$$

Torques in the null-space will not *accelerate* the task-space

Khatib, O. 1987. "A Unified Approach for Motion and Force Control of Robot Manipulators: The Operational Space Formulation." *IEEE Journal on Robotics and Automation* 3 (1). ieeexplore.ieee.org: 43–53.

## Transformation from joint-space to task space is power conserving:




$$\dot{\mathbf{q}}^T \tau = \dot{\mathbf{x}}^T \mathbf{F}$$

Dietrich, A., C. Ott, and S. Stramigioli. 2016. "Passivation of Projection-Based Null Space Compliance Control Via Energy Tanks." *IEEE Robotics and Automation Letters* 1 (1). ieeexplore.ieee.org: 184–91.

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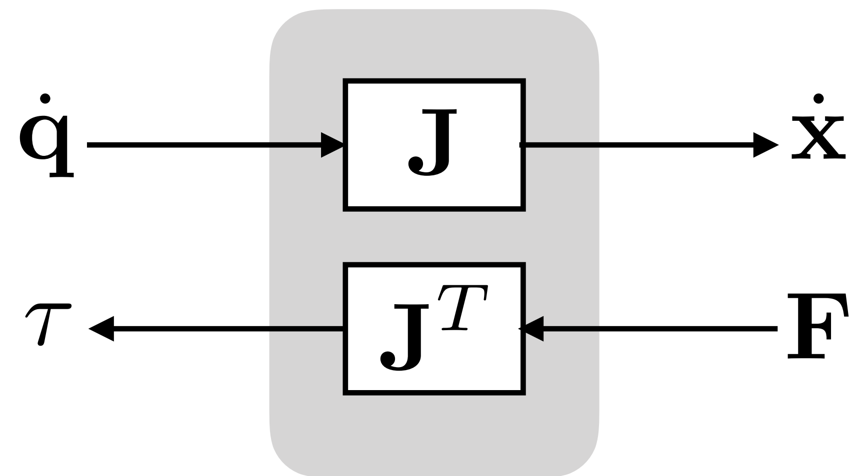
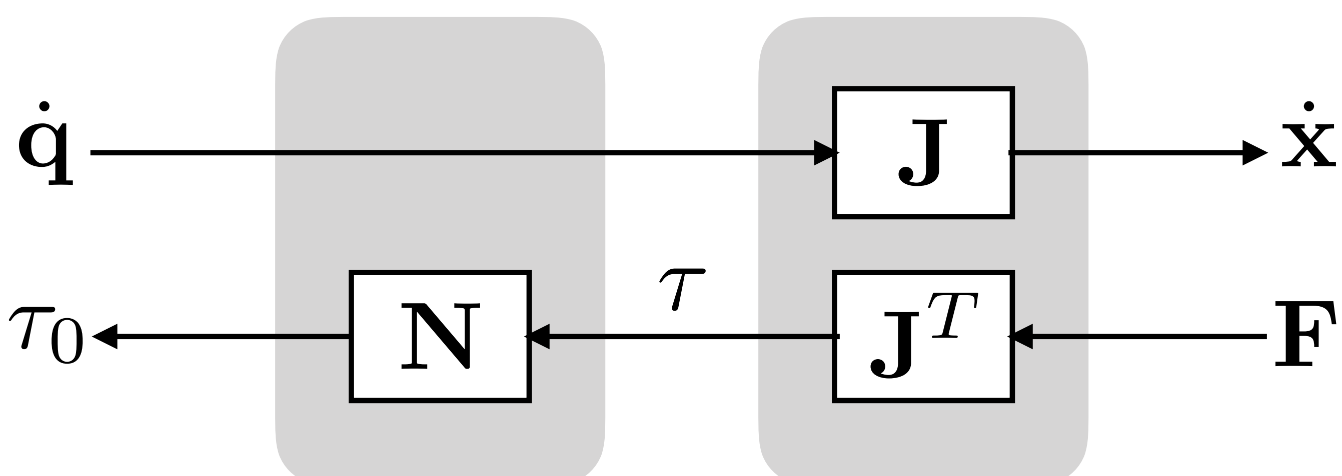



$$\dot{\mathbf{q}}^T \tau = \dot{\mathbf{x}}^T \mathbf{F}$$

**However, null-space torque transformations have no velocity dual, and therefore are NOT power conserving. We cannot guarantee passivity!**

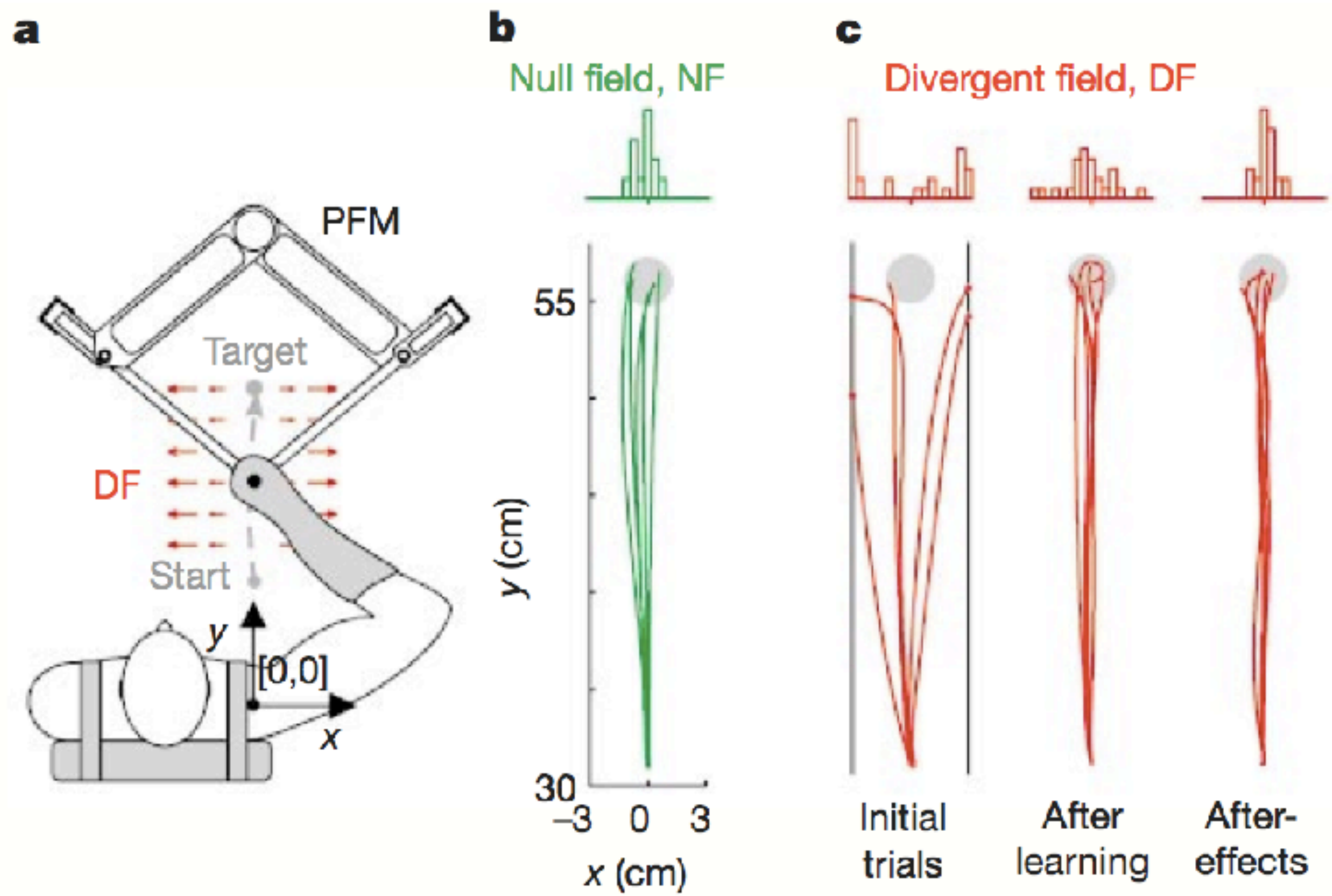



$$\dot{\mathbf{q}}^T \tau \neq \dot{\mathbf{q}}^T \tau_0$$

**Solution is to impose "virtual energy tanks" that absorb excess energy. Performance of secondary tasks will be impaired, but passivity preserved.**


Dietrich, A., C. Ott, and S. Stramigioli. 2016. "Passivation of Projection-Based Null Space Compliance Control Via Energy Tanks." *IEEE Robotics and Automation Letters* 1 (1). ieeexplore.ieee.org: 184–91.

# Cartesian Impedance Control in Humans




Burdet, Osu, Franklin, Milner, Kawato. The central nervous system stabilizes unstable dynamics by learning optimal impedance, Nature, Vol 414, 2001

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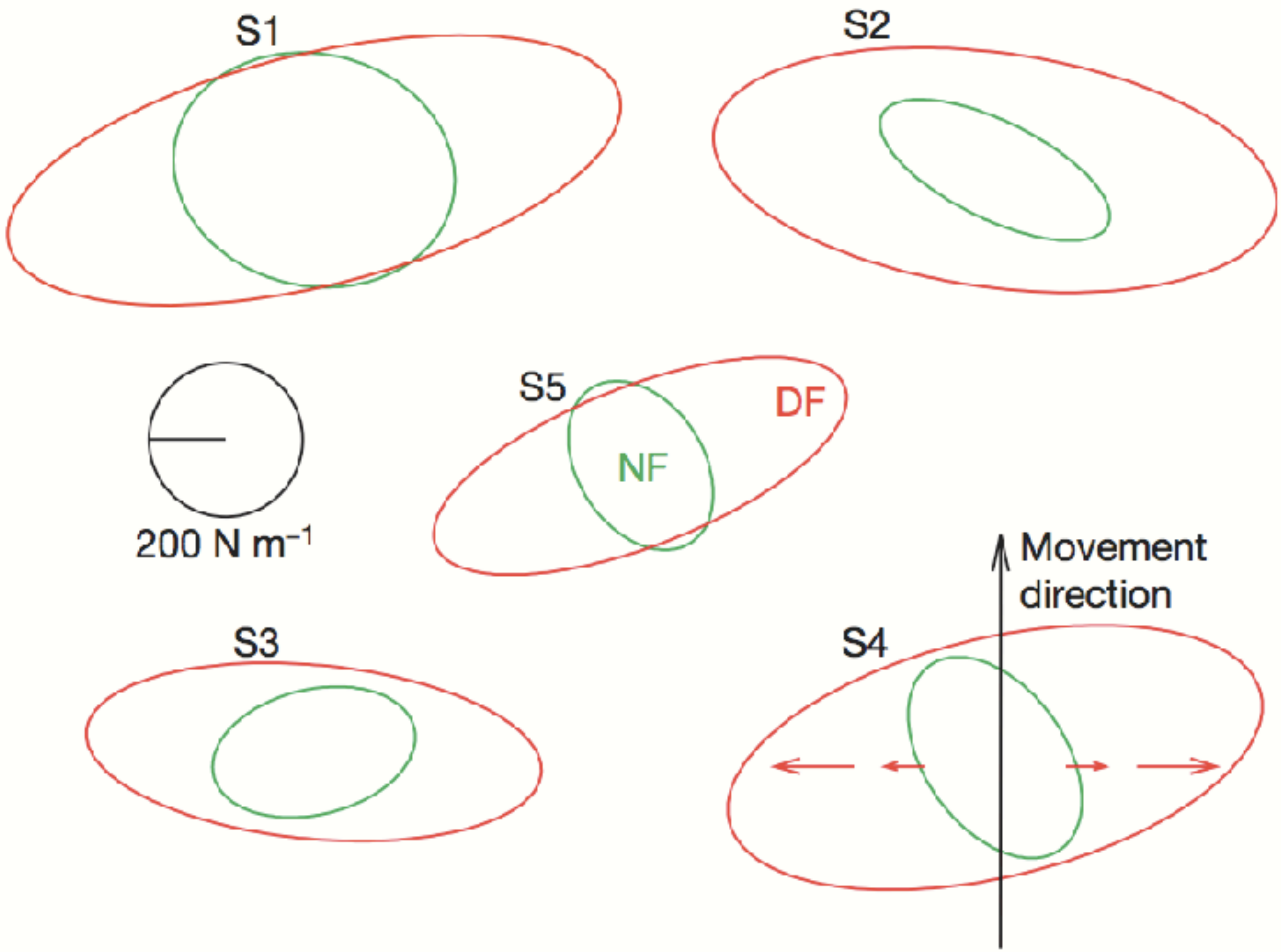


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## Cartesian Impedance Control

**We wish to impose the following impedance behaviour:**

$$\boldsymbol{\Lambda}_d\ddot{\tilde{\mathbf{x}}} + \mathbf{D}_d\dot{\tilde{\mathbf{x}}} + \mathbf{K}_d\tilde{\mathbf{x}} = \mathbf{F}_x$$

## Cartesian Impedance Control

**We wish to impose the following impedance behaviour:**

**Desired Damping Matrix**

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_d$$

**displacement from virtual set point**

$$\mathbf{\Lambda}_d \ddot{\tilde{\mathbf{x}}} + \mathbf{D}_d \dot{\tilde{\mathbf{x}}} + \mathbf{K}_d \tilde{\mathbf{x}} = \mathbf{F}_x$$

**Force at end-effector**

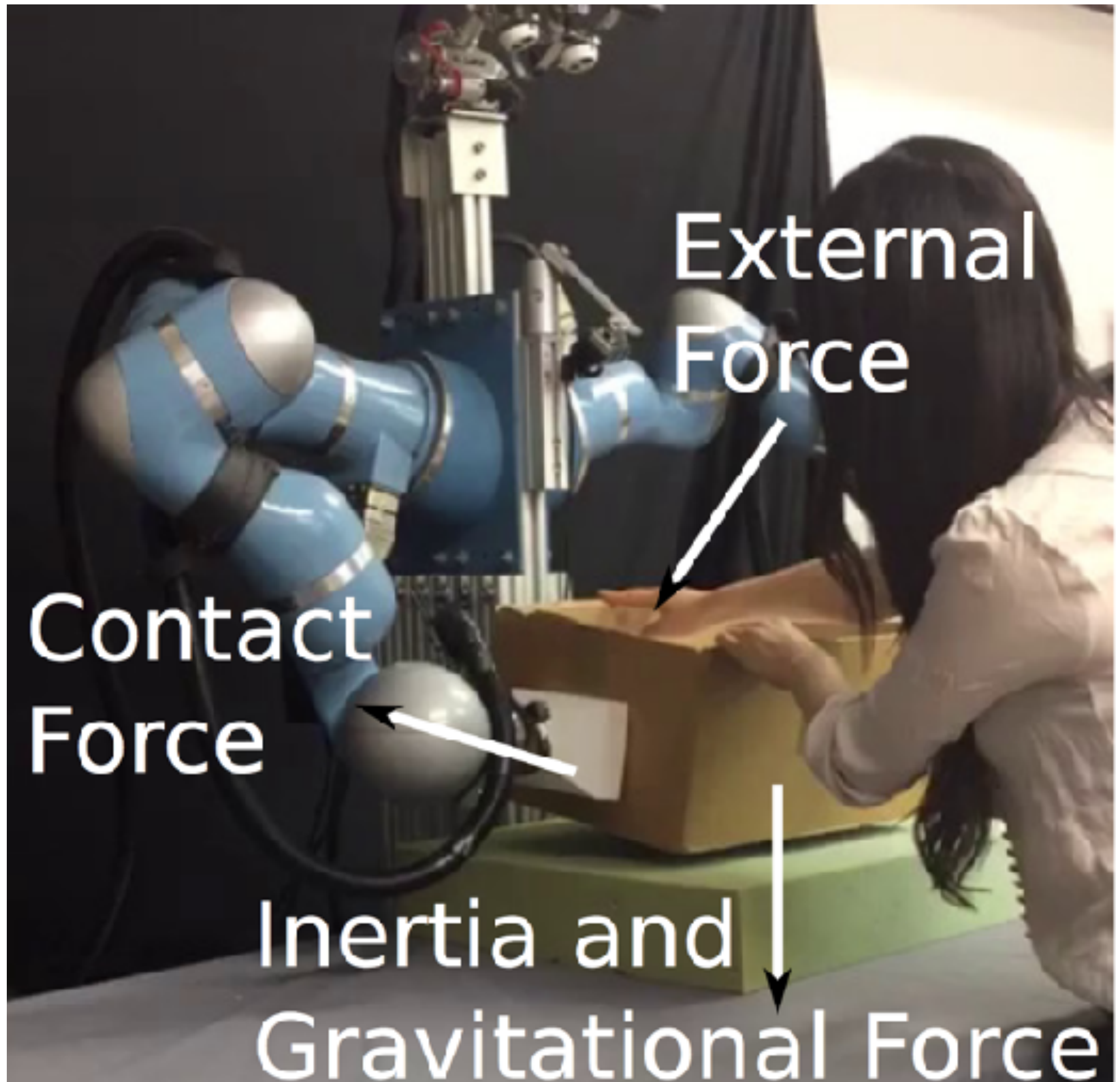
**Desired Inertia Matrix**

**Desired Stiffness Matrix**

# Dual-arm robot manipulating a rigid object

**We wish to impose 6-DOF cartesian impedance behaviour on the object:**

$$\mathbf{\Lambda}_d\ddot{\tilde{\mathbf{x}}} + \mathbf{D}_d\dot{\tilde{\mathbf{x}}} + \mathbf{K}_d\tilde{\mathbf{x}} = \mathbf{F}_x$$

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**Dynamics equation:**

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{h} = \tau + \mathbf{J}_c^T \lambda_c + \mathbf{J}_x^T \mathbf{F}_x$$

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- $\mathbf{M}$: **Inertia Matrix (dual-arm)**
- $\ddot{\mathbf{q}}$: **joint-acc (dual-arm)**
- $\mathbf{h}$: **gravity (dual-arm)**
- $\tau$: **actuator torque (dual-arm)**
- $\lambda_c$: **contact wrenches of robot hands on object**
- $\mathbf{F}_x$: **external wrenches on object (e.g. from human)**

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- $\mathbf{h}$: gravity (dual-arm)
- $\tau$: actuator torque (dual-arm)
- $\lambda_c$: contact wrenches of robot hands on object
- $\mathbf{F}_x$: external wrenches on object (e.g. from human)

$\lambda_c$ **produces no motion, only internal forces (we assume object is rigid)**

$\mathbf{F}_x$ **produces all the motion (and therefore is the output port of our impedance)**

$$\mathbf{J}_c^T = \begin{bmatrix} \mathbf{J}_L^T & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_R^T \end{bmatrix} \left(\mathbf{I} - \mathbf{G}^+\mathbf{G}\right)$$

**Jacobian is in the null-space of the Grasp Map (i.e. set of wrenches that produce no net wrench on the object**

Lin, Hsiu-Chin, Joshua Smith, Keyhan Kouhkiloui Babarahmati, Niels Dehio, and Michael Mistry. 2017. "A Projected Inverse Dynamics Approach for Dual-Arm Cartesian Impedance Control." *arXiv [cs.RO]*. arXiv. http://arxiv.org/abs/1707.00484.

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$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{h} = \tau + \mathbf{J}_c^T \lambda_c + \mathbf{J}_x^T \mathbf{F}_x$$

$\lambda_c$ **imposes no motion to the object, or force upon the human: it does not factor into our desired Impedance behaviour. We can get rid of it!**

$$\mathbf{P} = \mathbf{I} - \mathbf{J}_c^+ \mathbf{J}_c$$

**create orthogonal projection matrix in the null-space of the constraint Jacobian, and project out** $\lambda_c$

$$\mathbf{P}\mathbf{M}\ddot{\mathbf{q}} + \mathbf{P}\mathbf{h} = \mathbf{P}\tau + \mathbf{P}\mathbf{J}_x^T \mathbf{F}_x$$

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$$\mathbf{P}\mathbf{M}\ddot{\mathbf{q}} + \mathbf{P}\mathbf{h} = \mathbf{P}\tau + \mathbf{P}\mathbf{J}_x^T \mathbf{F}_x$$

**and its orthogonal sub-space defines the following constraint:**

$$(\mathbf{I} - \mathbf{P})\,\dot{\mathbf{q}} = 0$$
$$(\mathbf{I} - \mathbf{P})\,\ddot{\mathbf{q}} = \dot{\mathbf{P}}\dot{\mathbf{q}}$$

**no motion of the hands, relative to each other**

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**no motion of the hands, relative to each other**

**because P and (I-P) are orthogonal spaces, we can add them together:**

$$\mathbf{M}_c\ddot{\mathbf{q}} + \mathbf{P}\mathbf{h} - \dot{\mathbf{P}}\dot{\mathbf{q}} = \mathbf{P}\tau + \mathbf{P}\mathbf{J}_x^T \mathbf{F}_x$$
$$\mathbf{M}_c = \mathbf{P}\mathbf{M} + \mathbf{I} - \mathbf{P}$$

$$\mathbf{M}_c\ddot{\mathbf{q}} + \mathbf{P}\mathbf{h} - \dot{\mathbf{P}}\dot{\mathbf{q}} = \mathbf{P}\tau + \mathbf{P}\mathbf{J}_x^T\mathbf{F}_x$$

$$\mathbf{M}_c = \mathbf{P}\mathbf{M} + \mathbf{I} - \mathbf{P}$$ **is invertable!**


Aghili, F. 2005. "A Unified Approach for Inverse and Direct Dynamics of Constrained Multibody Systems Based on Linear Projection Operator: Applications to Control and Simulation." *IEEE Transactions on Robotics* 21 (5). ieeexplore.ieee.org: 834–49.

Mistry, Michael, and Ludovic Righetti. 2012. "Operational Space Control of Constrained and Underactuated Systems." *Robotics: Science and Systems VII*, 225–32.

$$\mathbf{M}_c\ddot{\mathbf{q}} + \mathbf{P}\mathbf{h} - \dot{\mathbf{P}}\dot{\mathbf{q}} = \mathbf{P}\tau + \mathbf{P}\mathbf{J}_x^T\mathbf{F}_x$$

$$\mathbf{M}_c = \mathbf{P}\mathbf{M} + \mathbf{I} - \mathbf{P}$$ **is invertable!**

**Multiply by:** $\mathbf{J}_x\mathbf{M}_c^{-1}$

$$\mathbf{J}_x\ddot{\mathbf{q}} + \mathbf{J}_x\mathbf{M}_c^{-1}\left(\mathbf{P}\mathbf{h} - \dot{\mathbf{P}}\dot{\mathbf{q}}\right) = \mathbf{J}_x\mathbf{M}_c^{-1}\mathbf{P}\tau + \mathbf{J}_x\mathbf{M}_c^{-1}\mathbf{P}\mathbf{J}_x^T\mathbf{F}_x$$


Aghili, F. 2005. “A Unified Approach for Inverse and Direct Dynamics of Constrained Multibody Systems Based on Linear Projection Operator: Applications to Control and Simulation.” *IEEE Transactions on Robotics* 21 (5). ieeexplore.ieee.org: 834–49.

Mistry, Michael, and Ludovic Righetti. 2012. “Operational Space Control of Constrained and Underactuated Systems.” *Robotics: Science and Systems VII*, 225–32.

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$$\mathbf{J}_x\ddot{\mathbf{q}} + \mathbf{J}_x\mathbf{M}_c^{-1}\left(\mathbf{P}\mathbf{h} - \dot{\mathbf{P}}\dot{\mathbf{q}}\right) = \mathbf{J}_x\mathbf{M}_c^{-1}\mathbf{P}\tau + \mathbf{J}_x\mathbf{M}_c^{-1}\mathbf{P}\mathbf{J}_x^T\mathbf{F}_x$$

**Replace:** $\mathbf{J}_x\ddot{\mathbf{q}} = \ddot{\mathbf{x}} - \dot{\mathbf{J}}_x\dot{\mathbf{q}}$

**Define:** $\mathbf{\Lambda}_c = \left(\mathbf{J}_x\mathbf{M}_c^{-1}\mathbf{P}\mathbf{J}_x^T\right)^{-1}$

**Cartesian (Constrained) Inertia Matrix**

**And multiply by** $\mathbf{\Lambda}_c$

$$\mathbf{\Lambda}_c\ddot{\mathbf{x}} + \mathbf{\Lambda}_c\mathbf{J}_x\mathbf{M}_c^{-1}\left(\mathbf{P}\mathbf{h} - \dot{\mathbf{P}}\dot{\mathbf{q}}\right) - \mathbf{\Lambda}_c\dot{\mathbf{J}}_x\dot{\mathbf{q}} = \mathbf{\Lambda}_c\mathbf{J}_x\mathbf{M}_c^{-1}\mathbf{P}\tau + \mathbf{F}_x$$


Aghili, F. 2005. “A Unified Approach for Inverse and Direct Dynamics of Constrained Multibody Systems Based on Linear Projection Operator: Applications to Control and Simulation.” *IEEE Transactions on Robotics* 21 (5). ieeexplore.ieee.org: 834–49.

Mistry, Michael, and Ludovic Righetti. 2012. “Operational Space Control of Constrained and Underactuated Systems.” *Robotics: Science and Systems VII*, 225–32.

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**Replace:** $\tau = \mathbf{J}_x^T\mathbf{F}$

**and lump all velocity terms into:** $\mathbf{h}_c$

**we have our operational space dynamics:** $\mathbf{\Lambda}_c\ddot{\mathbf{x}} + \mathbf{h}_c = \mathbf{F} + \mathbf{F}_x$

Lin, Hsiu-Chin, Joshua Smith, Keyhan Kouhkiloui Babarahmati, Niels Dehio, and Michael Mistry. 2017. "A Projected Inverse Dynamics Approach for Dual-Arm Cartesian Impedance Control." *arXiv [cs.RO]*. arXiv. http://arxiv.org/abs/1707.00484.

$$\boldsymbol{\Lambda}_c\ddot{\mathbf{x}} + \boldsymbol{\Lambda}_c\mathbf{J}_x\mathbf{M}_c^{-1}\left(\mathbf{P}\mathbf{h} - \dot{\mathbf{P}}\dot{\mathbf{q}}\right) - \boldsymbol{\Lambda}_c\dot{\mathbf{J}}_x\dot{\mathbf{q}} = \boldsymbol{\Lambda}_c\mathbf{J}_x\mathbf{M}_c^{-1}\mathbf{P}\tau + \mathbf{F}_x$$

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**and lump all velocity terms into:** $\mathbf{h}_c$

**we have our operational space dynamics:** $\boldsymbol{\Lambda}_c\ddot{\mathbf{x}} + \mathbf{h}_c = \mathbf{F} + \mathbf{F}_x$

**set F to the following controller:**

$$\mathbf{F} = \mathbf{h}_c + \boldsymbol{\Lambda}_c\ddot{\mathbf{x}}_d - \boldsymbol{\Lambda}_c\boldsymbol{\Lambda}_d^{-1}\left(\mathbf{D}_d\dot{\tilde{\mathbf{x}}} + \mathbf{K}_d\tilde{\mathbf{x}}\right) + \left(\boldsymbol{\Lambda}_c\boldsymbol{\Lambda}_d^{-1} - \mathbf{I}\right)\mathbf{F}_x$$

**and our desired impedance is achieved!**

$$\boldsymbol{\Lambda}_d\ddot{\tilde{\mathbf{x}}} + \mathbf{D}_d\dot{\tilde{\mathbf{x}}} + \mathbf{K}_d\tilde{\mathbf{x}} = \mathbf{F}_x$$

Lin, Hsiu-Chin, Joshua Smith, Keyhan Kouhkiloui Babarahmati, Niels Dehio, and Michael Mistry. 2017. "A Projected Inverse Dynamics Approach for Dual-Arm Cartesian Impedance Control." *arXiv [cs.RO]*. arXiv. http://arxiv.org/abs/1707.00484.

**Final impedance Controller:**

$$\mathbf{F} = \mathbf{h}_c + \mathbf{\Lambda}_c \ddot{\mathbf{x}}_d - \mathbf{\Lambda}_c \mathbf{\Lambda}_d^{-1} \left( \mathbf{D}_d \dot{\tilde{\mathbf{x}}} + \mathbf{K}_d \tilde{\mathbf{x}} \right) + \left( \mathbf{\Lambda}_c \mathbf{\Lambda}_d^{-1} - \mathbf{I} \right) \mathbf{F}_x$$

**However, controller requires knowledge of external forces e.g. through a Force/Torque sensor**

**Also: if we desire a small inertia, relative to actual inertia, Fx is effectively magnified**

**Can be dangerous, but more over is not passive!**

Lin, Hsiu-Chin, Joshua Smith, Keyhan Kouhkiloui Babarahmati, Niels Dehio, and Michael Mistry. 2017. "A Projected Inverse Dynamics Approach for Dual-Arm Cartesian Impedance Control." *arXiv [cs.RO]*. arXiv. http://arxiv.org/abs/1707.00484.

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**However, controller requires knowledge of external forces e.g. through a Force/Torque sensor**

**Also: if we desire a small inertia, relative to actual inertia, Fx is effectively magnified**

**Can be dangerous, but more over is not passive!**

**Controller without inertia shaping, set:** $\mathbf{\Lambda}_d = \mathbf{\Lambda}_c$

**Control equation simplifies to:**

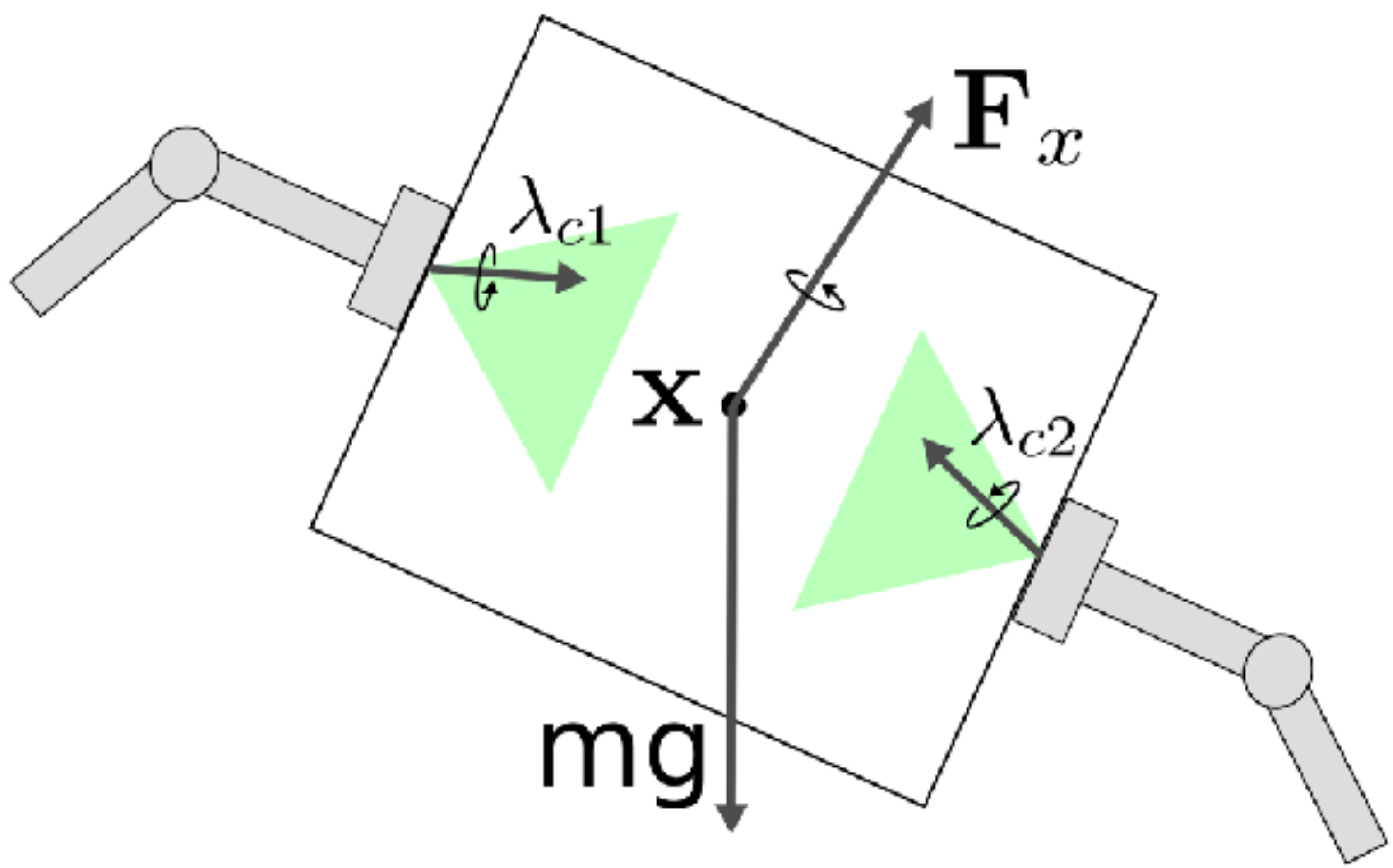
$$\mathbf{F} = \mathbf{h}_c + \mathbf{\Lambda}_c \ddot{\mathbf{x}}_d - \mathbf{D}_d \dot{\tilde{\mathbf{x}}} - \mathbf{K}_d \tilde{\mathbf{x}}$$

**and is passive again.** **Also no contact force sensor required to measure Fx!**

Lin, Hsiu-Chin, Joshua Smith, Keyhan Kouhkiloui Babarahmati, Niels Dehio, and Michael Mistry. 2017. "A Projected Inverse Dynamics Approach for Dual-Arm Cartesian Impedance Control." *arXiv [cs.RO]*. arXiv. http://arxiv.org/abs/1707.00484.

So far, we showed how to regulate the impedance behaviour of the object.

But how to regulate the internal forces, e.g. to prevent slipping or dropping the object?




given uncertain Fx, how can we be sure contact wrenches will stay inside their friction cones?

**equation of motion:** $$\mathbf{P}\mathbf{M}\ddot{\mathbf{q}} + \mathbf{P}\mathbf{h} = \mathbf{P}\tau + \mathbf{P}\mathbf{J}_x^T\mathbf{F}_x$$

**equation of internal forces:**

$$\left(\mathbf{I} - \mathbf{P}\right)\mathbf{M}\ddot{\mathbf{q}} + \mathbf{h} = \left(\mathbf{I} - \mathbf{P}\right)\left(\tau + \mathbf{J}_x^T\mathbf{F}_x\right) + \mathbf{J}_c^T\lambda_c$$

equation of motion: $$\mathbf{PM}\ddot{\mathbf{q}} + \mathbf{Ph} = \mathbf{P}\tau + \mathbf{PJ}_x^T\mathbf{F}_x$$

equation of internal forces:

$$(\mathbf{I} - \mathbf{P})\,\mathbf{M}\ddot{\mathbf{q}} + \mathbf{h} = (\mathbf{I} - \mathbf{P})\left(\tau + \mathbf{J}_x^T\mathbf{F}_x\right) + \mathbf{J}_c^T\lambda_c$$

formulate an optimisation:

$$\underset{\boldsymbol{\tau}}{\text{minimise}}\ (\mathbf{I} - \mathbf{P})\boldsymbol{\tau}$$

$$\text{subject to}\ \lambda_{f,z}^i \geq 0$$ unilateral

$$\mu\lambda_{f,z}^i \geq \sqrt{(\lambda_{f,x}^i)^2 + (\lambda_{f,y}^i)^2}$$ friction cone

$$\gamma\lambda_{f,z}^i \geq |\lambda_{m,z}^i|$$ torsional friction

$$\delta x\lambda_{f,z}^i \geq |\lambda_{m,x}^i|$$ COPx

$$\delta y\lambda_{f,z}^i \geq |\lambda_{m,y}^i|$$ COPy

Note: requires knowledge of Fx. Do we need a force sensor?

equation of motion: $\mathbf{P}\mathbf{M}\ddot{\mathbf{q}} + \mathbf{P}\mathbf{h} = \mathbf{P}\tau + \mathbf{P}\mathbf{J}_x^T\mathbf{F}_x$

equation of internal forces:

$$(\mathbf{I} - \mathbf{P})\,\mathbf{M}\ddot{\mathbf{q}} + \mathbf{h} = (\mathbf{I} - \mathbf{P})\left(\tau + \mathbf{J}_x^T\mathbf{F}_x\right) + \mathbf{J}_c^T\lambda_c$$

formulate an optimisation:

$$\begin{aligned}
\underset{\boldsymbol{\tau}}{\text{minimise}}\ & (\mathbf{I} - \mathbf{P})\boldsymbol{\tau} \\
\text{subject to}\ & \lambda_{f,z}^i \geq 0 && \text{unilateral} \\
& \mu\lambda_{f,z}^i \geq \sqrt{(\lambda_{f,x}^i)^2 + (\lambda_{f,y}^i)^2} && \text{friction cone} \\
& \gamma\lambda_{f,z}^i \geq |\lambda_{m,z}^i| && \text{torsional friction} \\
& \delta x\lambda_{f,z}^i \geq |\lambda_{m,x}^i| && \text{COPx} \\
& \delta y\lambda_{f,z}^i \geq |\lambda_{m,y}^i| && \text{COPy}
\end{aligned}$$

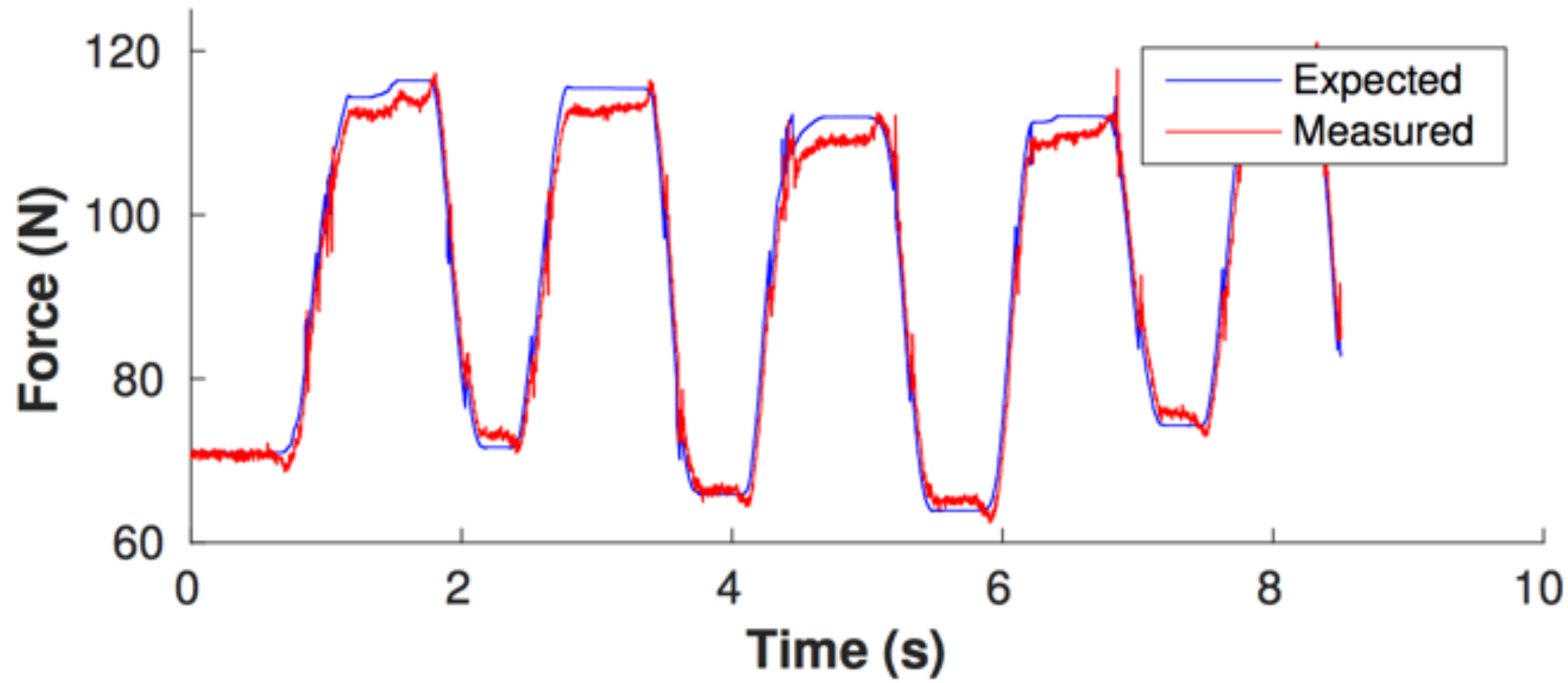
Note: requires knowledge of Fx. Do we need a force sensor?

No! we can estimate Fx from our Impedance behaviour! $\mathbf{\Lambda}_d\ddot{\tilde{\mathbf{x}}} + \mathbf{D}_d\dot{\tilde{\mathbf{x}}} + \mathbf{K}_d\tilde{\mathbf{x}} = \mathbf{F}_x$

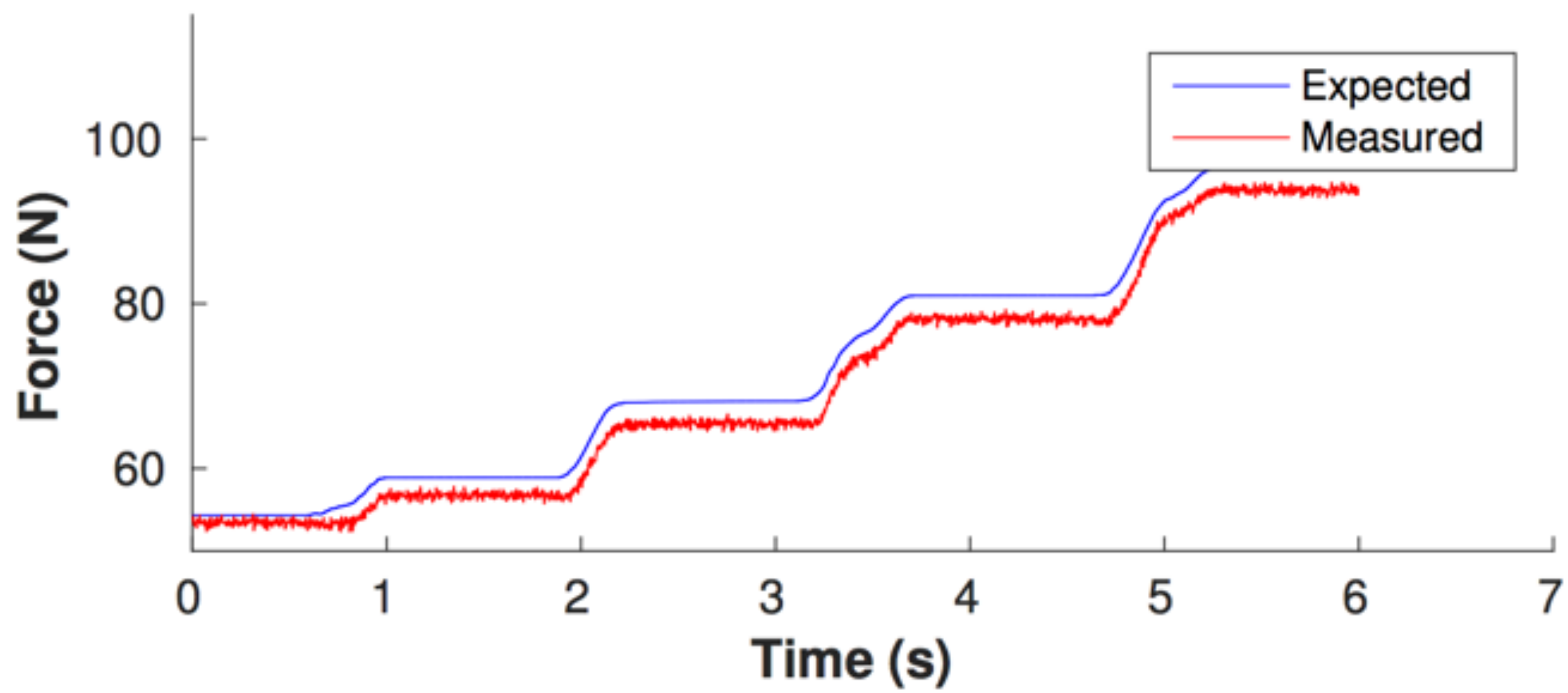
# Experiment 2
# Dual Arm Holding an Object

Lin, Hsiu-Chin, Joshua Smith, Keyhan Kouhkiloui Babarahmati, Niels Dehio, and Michael Mistry. 2017. "A Projected Inverse Dynamics Approach for Dual-Arm Cartesian Impedance Control." *arXiv [cs.RO]*. arXiv. http://arxiv.org/abs/1707.00484.

# Internal Forces (squeezing object) as external forces vary



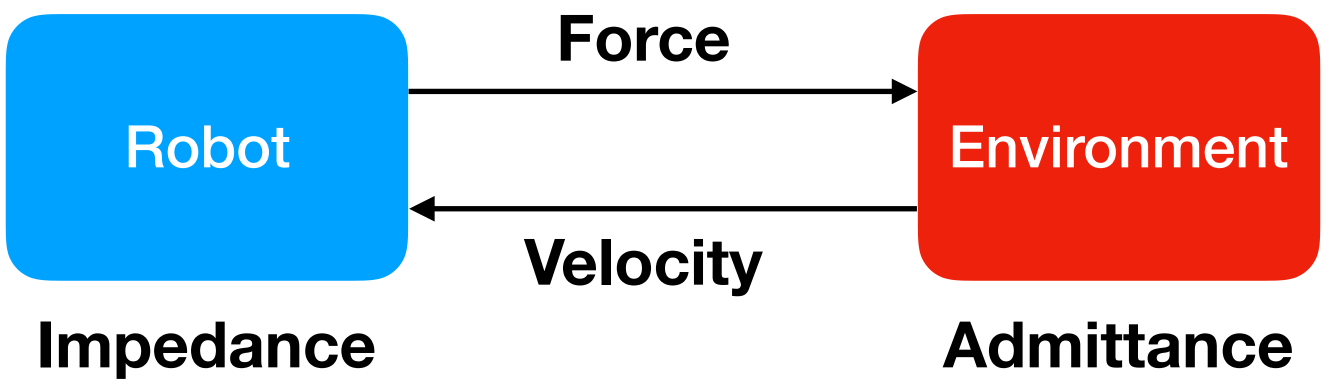

human interaction




adding mass

Lin, Hsiu-Chin, Joshua Smith, Keyhan Kouhkiloui Babarahmati, Niels Dehio, and Michael Mistry. 2017. “A Projected Inverse Dynamics Approach for Dual-Arm Cartesian Impedance Control.” *arXiv [cs.RO]*. arXiv. http://arxiv.org/abs/1707.00484.

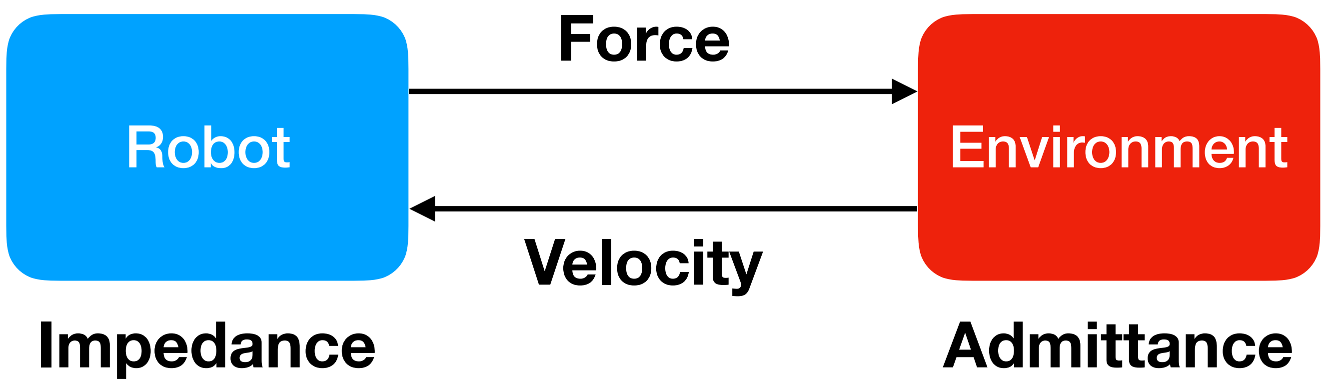
## Conclusion




**As the environment is typically an admittance, it is best (safest) to treat the robot as an impedance**

**Moreover, if the robot is a passive impedance, we can maintain safe and stable interaction given uncertain environment, models, etc.**

## Conclusion




As the environment is typically an admittance, it is best (safest) to treat the robot as an impedance

Moreover, if the robot is a passive impedance, we can maintain safe and stable interaction given uncertain environment, models, etc.

Caveat! But only if the environment is passive too!

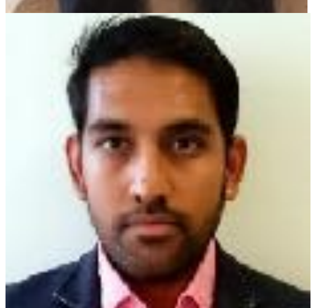
Humans are not passive! (i.e. we are able to generate energy)

However, it is generally agreed that a passive robot is the safest way to interact with humans

# Thanks to:

Morteza Azad

Hsiu-Chin Lin



Vijaykumar Rajasekaran

Chie Takahashi

Joshua Smith

Valerio Ortenzi

Keyhan Kouhkiloui

Maxime Adjigble

Michael Jacob Mathews

NDA

EPSRC
Engineering and Physical Sciences
Research Council

CODYCO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